GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP2

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP2 Paper B – Marking Guide

1. $\dfrac{-y}{\sqrt{1-x^2}} + \arccos x \dfrac{dy}{dx} - \frac{1}{\pi} e^{2x} - \frac{2x}{\pi} e^{2x} = 0$ M1 A2

when $x = 0$, $y \times \frac{\pi}{2} - 0 - 1 = 0$ $\therefore$ $y = \frac{2}{\pi}$ B1

when $x = 0$, $-\frac{2}{\pi} + \frac{\pi}{2}\dfrac{dy}{dx} - \frac{1}{\pi} - 0 = 0$ M1 A1

$\therefore$ $\frac{\pi}{2}\dfrac{dy}{dx} = \frac{3}{\pi}$ so $\dfrac{dy}{dx} = \frac{6}{\pi^2}$ A1 **(7)**

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2. $f'(x) = 5\sinh x + 3\cosh x$ M1

S.P. $\therefore$ $5\sinh x + 3\cosh x = 0$ giving $\tanh x = -\frac{3}{5}$ M1 A1

$x = \operatorname{artanh}(-\frac{3}{5}) = \frac{1}{2}\ln\left(\dfrac{1-\frac{3}{5}}{1+\frac{3}{5}}\right)$ M1 A1

$x = \frac{1}{2}\ln\frac{1}{4} = -\ln 2$ A1

$f(-\ln 2) = 5\cosh(-\ln 2) + 3\sinh(-\ln 2) = 4$ M1

$\therefore$ $p = -1$, $q = 2$, $r = 4$ A1 **(8)**

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3. *(a)* $x(mx + c) = -9$ $\therefore$ $mx^2 + cx + 9 = 0$ M1 A1

tangent $\therefore$ "$b^2 - 4ac$" $= 0$ $\therefore$ $c^2 - 4 \times m \times 9 = 0$ M1

$\therefore$ $c^2 = 36m$ giving $c = \pm 6\sqrt{m}$ A1

(b) $(4, -2)$ $\therefore$ $-2 = 4m + c$ and $c^2 = 36m$

$\therefore$ $(-2 - 4m)^2 = 36m$ M1

$4 + 16m + 16m^2 = 36m$

giving $4m^2 - 5m + 1 = 0$

$(4m - 1)(m - 1) = 0$ M1

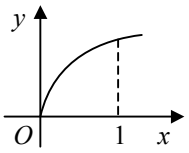
$m = \frac{1}{4}$ or 1 A1

if $m = \frac{1}{4}$, $c = -3$; if $m = 1$, $c = -6$ M1

$\therefore$ tangents are $y = \frac{1}{4}x - 3$ and $y = x - 6$ A1 **(9)**

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4. $y^2 = x \ \therefore\ 2y\dfrac{dy}{dx} = 1$ so $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$ M1 A1



$\therefore$ lower limit = 0 M1

$A = \int_0^1 2\pi y\sqrt{1+\frac{1}{4x}}\ dx$ M1 A1

$= \int_0^1 2\pi\sqrt{x}\ \sqrt{1+\frac{1}{4x}}\ dx = \int_0^1 \pi\sqrt{4x+1}\ dx$ M1 A1

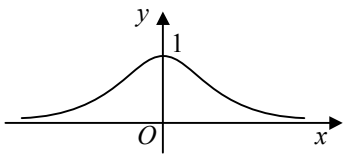
$= \pi[\frac{2}{3}\times\frac{1}{4}(4x+1)^{\frac{3}{2}}]_0^1$ M1 A1

$= \frac{1}{6}\pi[5^{\frac{3}{2}} - 1^{\frac{3}{2}}] = \frac{1}{6}\pi(5\sqrt{5}-1)$ M1 A1 **(11)**

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5. *(a)* $\cosh x = \frac{1}{2}(e^x + e^{-x})$ $\therefore$ $\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$ B1

(b)



B2

(c) $\int \operatorname{sech} x\ dx = \int \dfrac{2}{e^x + e^{-x}}\ dx = \int \dfrac{2e^x}{e^{2x}+1}\ dx$

$u = e^x$ $\therefore$ $\dfrac{du}{dx} = e^x$ M1

$I = \int \dfrac{2}{u^2+1}\ du$ A1

$= 2\arctan u + c = 2\arctan e^x + c$ M1 A1

(d) $V = \int_{-a}^{a} \pi\operatorname{sech}^2 x\ dx$ M1

$= [\pi\tanh x]_{-a}^{a}$ A1

$= \pi[\tanh a - \tanh(-a)] = 2\pi\tanh a$ M1 A1

(e) as $a \to \infty$, $\tanh a \to 1$, $V \to 2\pi$ $\therefore$ limit of volume is 2π A1 **(12)**

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6. *(a)* $u = (2 - x^2)^n,\ u' = -2nx(2 - x^2)^{n-1};\ \ v' = 1,\ v = x$ M1

$I_n = [x(2 - x^2)^n]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} -2nx^2(2 - x^2)^{n-1}\ \mathrm{d}x$ A1

$I_n = [0 - 0] - 2n\int_0^{\sqrt{2}} (-x^2 + 2 - 2)(2 - x^2)^{n-1}\ \mathrm{d}x$ M2 A1

$I_n = -2n\int_0^{\sqrt{2}} (2 - x^2)(2 - x^2)^{n-1}\ \mathrm{d}x - 2n\int_0^{\sqrt{2}} -2(2 - x^2)^{n-1}\ \mathrm{d}x$ M1

$I_n = -2n\int_0^{\sqrt{2}} (2 - x^2)^n\ \mathrm{d}x + 4n\int_0^{\sqrt{2}} (2 - x^2)^{n-1}\ \mathrm{d}x$ A1

$I_n = -2nI_n + 4nI_{n-1}$ M1

$(1 + 2n)I_n = 4nI_{n-1}$

$I_n = \frac{4n}{2n+1} I_{n-1}$ A1

(b) $I_0 = \int_0^{\sqrt{2}} \mathrm{d}x = [x]_0^{\sqrt{2}} = \sqrt{2}$ B1

$I_1 = \frac{4}{3} I_0 = \frac{4}{3}\sqrt{2}$ M1

$I_2 = \frac{8}{5} I_1 = \frac{8}{5} \times \frac{4}{3}\sqrt{2}$ M1

$I_3 = \frac{12}{7} I_2 = \frac{12}{7} \times \frac{8}{5} \times \frac{4}{3}\sqrt{2} = \frac{128}{35}\sqrt{2}$ A1 **(13)**

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7. *(a)* $\rho = \dfrac{\mathrm{d}s}{\mathrm{d}\psi} = \dfrac{\frac{1}{2}\sec^2 \frac{1}{2}\psi}{\tan \frac{1}{2}\psi}$ M1 A1

$= \frac{1}{2} \times \dfrac{1}{\cos^2 \frac{1}{2}\psi} \times \dfrac{\cos \frac{1}{2}\psi}{\sin \frac{1}{2}\psi} = \dfrac{1}{2\cos \frac{1}{2}\psi \sin \frac{1}{2}\psi} = \dfrac{1}{\sin \psi} = \operatorname{cosec} \psi$ M1 A1

(b) $\dfrac{\mathrm{d}s}{\mathrm{d}\psi} = \operatorname{cosec} \psi,\ \ \dfrac{\mathrm{d}y}{\mathrm{d}s} = \sin \psi$

$\dfrac{\mathrm{d}y}{\mathrm{d}\psi} = \dfrac{\mathrm{d}y}{\mathrm{d}s}\dfrac{\mathrm{d}s}{\mathrm{d}\psi} = 1 \ \ \therefore y = \psi + c$ M1 A1

$y = \frac{\pi}{2},\ \psi = \frac{\pi}{2} \ \ \therefore c = 0$ so $y = \psi$ M1 A1

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan \psi = \tan y$ M1

$\int_{\frac{\pi}{2}}^{y} \cot y\ \mathrm{d}y = \int_0^x \mathrm{d}x$ M1 A1

$[\ln |\sin y|]_{\frac{\pi}{2}}^{y} = [x]_0^x$ M1 A1

$\ln (\sin y) - \ln 1 = x - 0 \quad [0 < y \leq \frac{\pi}{2} \ \ \therefore \sin y > 0]$ M1

$x = \ln (\sin y)$ A1 **(15)**

Total **(75)**

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Performance Record – FP2 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	diff. inv. trig.	diff. hyp. fns., eqn. in hyp. fns.	hyperbola, tangent	surface area	integr. hyp. fns	reduction formula	intrinsic coords, rad. of curv.	
Marks	7	8	9	11	12	13	15	75
Student								

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